PHASE ANALYSIS OF SUPERRADIANCE OF A QUANTUM-DOT ENSEMBLE

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The phase analysis of the dynamic equations obtained in [13] on the basis of both a microscopic representation of the polarization of a quantum-dot ensemble and the difference of electron-level populations is carried out. It is shown that, under pumping and real relations between the parameters of a quantum-dot ensemble, the superradiance is realized in the form of a giant pulse regardless of the resonator frequency detuning and the coupling parameters. The obtained results are compared with experimental data.

1. Introduction

The phenomenon of superradiance in atoms and molecules is well known and thoroughly investigated (see, e.g., [1, 2]). Recently, superradiance was also observed for Bose–Einstein condensates of atoms [3], nuclear spins [4], and magnetic molecules [5]. In [6], it was assumed that, in the case where the distances between quantum dots or wells do not exceed the radiation length, they get involved in the effective interaction through the electromagnetic field, due to which nanostructures can pass also to the superradiant mode. The development of up-to-date technologies allows one to form ensembles of nanoobjects with a density sufficient to provide such a mode [7]. As a result, superradiance was discovered for quantum dots [8], semiconductor heterostructures [9], and photon crystals [10].

From the physical point of view, such a behavior is explained by the fact that the motion of charge carriers in quantum wells, wires, and dots is confined to one, two, and three directions, respectively. Due to that, the indicated nanoobjects are characterized by quantum energy levels typical of either isolated atoms or molecules (that is why quantum dots are called "artificial atoms") [11, 12]. Taking into account that, with increase in the dimensions of nanoobjects, the density of their electron states considerably decreases, so that the quantum properties are most pronounced in quantum dots and get much weaker in wires and wells.

A consistent theory of the superradiance of a quantum-dot ensemble based on a microscopic representation was developed in the recent work [13]. It was shown that the evolution of the system can be reduced to the following stages:

- *fluctuation mode* lasting during a short time interval of $10^{-15} \div 10^{-14}$ s, in which quantum dots representing electric dipoles autonomously emit electromagnetic waves but do not yet interact with one another;
- *quantum stage* lasting till the time moment $10^{-14} \div 10^{-13}$ s is characterized by the effective photon exchange between dipoles, but the coherence is still absent;
- *coherent stage* lasts till the time $t_{\mathrm{coh}} \sim 10^{-13} \div 10^{-12}$ s; quantum dots imitate a superradiance pulse (with a maximum reached at the end of the delay time interval of the order of $5t_{\mathrm{coh}}$, while its duration approximately equals $2t_{\mathrm{coh}}$);
- after the emission of the pulse of electromagnetic radiation, the system relaxes to a *non-coherent state* during the time $T \sim 10^{-9}$ s;
- at $t > T$, the quantum-dot ensemble not subjected to the external pumping or consisting of weakly interacting dipoles passes to the *stationary state* corresponding to the attracting node of the phase plane; otherwise, there arises a sequence consisting

of nearly 10 pulses with a period of the order of 10^{-13} s.

A peculiarity of the approach [13] consists in that the transition from microscopic to macroscopic quantities is performed with the use of two fundamentally different averaging procedures: in the framework of the first one, one searches for statistical states of the quantum system using the mean-field splitting of all correlators; in the following averaging of dynamic quantities over the stochastic variable, one keeps the even correlators of the amplitudes of their fluctuations. Such an approach allows one to successively consider collective effects resulting in a renormalization of the effective values of parameters of the system (in contrast to the superradiance of atoms and molecules, these effects play the main role in the ensemble of semiconductor nanoobjects).

The technique proposed in [13] not only describes the developed superradiant mode but also consistently reproduces the mechanism of its reaching due to the amplification of the electromagnetic radiation. In this case, the use of the single-mode approximation appears insufficient as the noncontradictory picture of the phenomenon requires to consider the total collection of the transverse radiation modes, over which the space averaging is to be performed. The procedure yielded the dynamic equations connecting the rates of change of the polarization of the quantum-dot ensemble P and the differences of the electron-level populations S with the values of P and S. The numerical solution of these equations shows that, at certain relations between the parameters, the system can generate both single pulses of electromagnetic radiation and their sequences. However, the values of parameters used in this case are not realized in semiconductor nanoobjects (for example, the nonmonotonous time dependences given in Fig. 3 of work [13] were obtained at the anomalously large attenuation parameter of the population difference γ_1 hereinafter denoted by γ_S). That is why a more detailed investigation of the possible modes of generation of electromagnetic radiation by a quantum-dot ensemble is considered urgent.

As is known from synergetics, the simplest way to perform such an investigation is to employ the phase-plane method that considers a self-consistent change of the quantities P and S (instead of their time dependences $P(t)$ and $S(t)$) expressed by the phase relation $P(S)$ [17–20]. The proposed work is devoted to such a study. Section 2 briefly describes the technique given in [13] that yields dynamic equations for the polarization P and the population difference S. In Section 3, these equations are used for the study of the conditions of superradiance. The conclusions about the possible modes of superradiance at real values of the parameters of a quantum-dot ensemble are presented in Section 4.

2. Statement of the Problem

According to [13], the microscopic behavior of a system is determined by the time dependences of the pseudospin operators $\sigma_i^-(t)$, $\sigma_i^+(t)$, and $\sigma_i^z(t)$ distributed over nodes $i = 1, \ldots, N$, the vector potential, and the intensities of the electromagnetic radiation field, as well the currents induced by quantum dots and their semiconductor environment. Moreover, the behavior of the pseudospins is specified by the Heisenberg equation, whereas the electromagnetic field obeys the Maxwell operator equations. Solving the latter, one can express the field potential in terms of the corresponding currents. As a result, it can be presented in the form of a sum of contributions caused by the self-action of dipole momenta of quantum dots, their radiation, and a stochastic component related to random changes of the fields of the dipoles and their environment. The use of this potential results in a closed system of equations for the pseudospins interacting through the electromagnetic radiation field.

The macroscopic behavior of the system is determined by the quantum averages of the radiation field intensity $E_i(t) = 2\left\langle \sigma_i^-(t) \right\rangle$, the polarization $P_i(t) = (2/N) \sum_{j(\neq i)} \left\langle \sigma_i^+(t)\sigma_j^-(t) + \sigma_i^-(t)\sigma_j^+(t) \right\rangle$, and the population difference $S_i(t) = 2\left\langle \sigma_i^z(t) \right\rangle$. As the distance between the quantum dots is much less than the radiation wavelength, one can pass from the summation over the nodes to the integration over the coordinate. On the other hand, the size of the medium the radiation passes through considerably exceeds its wavelength, that is why it is suitable to present it in the form of a cylinder with the z-axis parallel to the wave vector $\mathbf{k}$ of the radiation field. In this case, one can average all the space dependences over the direction normal to the cylinder axis and describe the longitudinal dependence of the radiation field by the plane wave $e^{i(kz-\omega t)}$ with frequency ω. As a result, the effective force acting on a quantum dot takes the form $f = Fe^{-i\omega t} + \xi(t)$, where the amplitude F determines the deterministic component and the term $\xi(t)$ denotes the stochastic contribution caused by random changes of the dipole fields and the semiconductor medium.

The equations of motion obtained due to the indicated transformations must be averaged over the stochastic additive $\xi(t)$. In this case, it is worth to consider the following fundamental fact [13]: whereas the quantum averaging marked above by the angle brackets supposes the splitting of the pseudospin correlators corresponding to different nodes, the averaging over the noise $\xi(t)$ must be performed, by assuming that $\overline{\xi^*(t)\xi(t')} \neq 0$ at $\overline{\xi(t)} = 0$ (hereinafter, the bar over symbols marks the averaging over $\xi(t)$). As a result, the equations of motion that describe the stochastic radiation of quantum dots take the form [13]

$$\frac{dE}{dt} = -\left[i\Omega(S) + \Gamma_P(S)\right]E + fS, \tag{1}$$

$$\frac{dP}{dt} = -2\Gamma_P(S)P + \left(f^*E + E^*f\right)S, \tag{2}$$

$$\frac{dS}{dt} = -\gamma_S\left(S - S_{\rm e}\right) - g_P\gamma_P P - \frac{1}{2}\left(f^*E + E^*f\right)S. \tag{3}$$

Here, the collective frequency of radiation $\Omega = \Omega(S)$ and the effective attenuation decrement $\Gamma_P = \Gamma_P(S)$ are determined by the expressions

$$\Omega \equiv \omega_0 + g_S\gamma_P S, \qquad \Gamma_P \equiv \gamma_P\left(1 - g_P S\right), \tag{4}$$

where ω_0 stands for the resonator's natural frequency, while γ_P, γ_S and g_P, g_S are the attenuation parameters and the coupling constants of the quantities P and S, respectively.

In system (1)–(3), the attention is attracted by the large factor $\Omega(S)$ on the right-hand side of Eq.(1). That is why assuming that

$$\frac{\gamma_S}{\omega_0} \ll 1, \quad \frac{\gamma_P}{\omega_0} \ll 1, \tag{5}$$

one can accept that the field $E(t)$ changes much faster than the polarization $P(t)$ and the population difference $S(t)$. This allows one to integrate Eq.(1), by assuming the two last quantities to be constant. As a result, we obtain the time dependence of the radiation field intensity in the form [13]

$$E = \left(E_0 - \frac{FS}{\delta + i\Gamma_P}\right)e^{-(i\Omega+\Gamma_P)t} + \frac{FS}{\delta + i\Gamma_P}e^{-i\omega t} +$$

$$+S\int_0^t \xi(t')e^{-(i\Omega+\Gamma_P)(t-t')}dt'. \tag{6}$$

Here, the quantity

$$\delta \equiv \omega - \Omega = \Delta - g_S\gamma_P S \tag{7}$$

is specified by the frequency deviation $\Delta \equiv \omega - \omega_0$.

The substitution of expression (6) into Eqs.(2) and (3) results in the appearance of the terms proportional to the correlator $\overline{\xi^*(t)\xi(t')}$ that quickly changes depending on the times t and t'. Averaging over the latter, one obtains the attenuation decrement

$$\gamma \equiv \Re \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\int_0^t \overline{\xi^*(t)\xi(t')}e^{-(i\Omega+\Gamma_P)(t-t')}dt'. \tag{8}$$

In this case, Eqs. (2) and (3) take the form

$$\frac{dP}{dt} = -2\gamma_P\left(1 - g_P S\right)P + 2\Gamma S^2, \tag{9}$$

$$\frac{dS}{dt} = \gamma_S S_{\rm e} - \left(\gamma_S + \Gamma\right)S - g_P\gamma_P P, \tag{10}$$

where the effective attenuation decrement

$$\Gamma \equiv \gamma + \frac{|F|^2\Gamma_P}{\Gamma_P^2 + \delta^2}\left(1 - {\rm e}^{-\Gamma_P t}\right) \simeq \gamma + \frac{|F|^2\Gamma_P}{\Gamma_P^2 + \delta^2}, \tag{11}$$

takes on a constant value at $t \gg \Gamma_P^{-1}$.

Analyzing system (9), (10), it is suitable to measure the time t in units of ω_0^{-1} and to relate the frequency ω, the attenuation decrements γ_P, γ_S, γ, and Γ and the field amplitude F to the natural frequency ω_0, and the polarization P to g_P^{-2}, whereas the population difference S is related to to g_P^{-1}. After that, Eqs. (9) and (10) take the simplified form

$$\frac{dP}{dt} = -2\gamma_P\left(1 - S\right)P + 2\Gamma S^2, \tag{12}$$

$$\frac{dS}{dt} = \gamma_S S_{\rm e} - \left(\gamma_S + \Gamma\right)S - \gamma_P P \tag{13}$$

containing the effective attenuation decrement

$$\Gamma \simeq \gamma + \frac{\gamma_P|F|^2\left(1 - S\right)}{\gamma_P^2\left(1 - S\right)^2 + \left[(\omega - 1) - g\gamma_P S\right]^2} \tag{14}$$

with the ratio of the coupling constants $g \equiv g_S/g_P \sim 1$.

3. Investigation of the Conditions of Superradiance

According to the technique given in [14], the solution of Eqs.(12) and (13) is determined by the stationary state $P = P_0$, $S = S_0$, in which the polarization and the

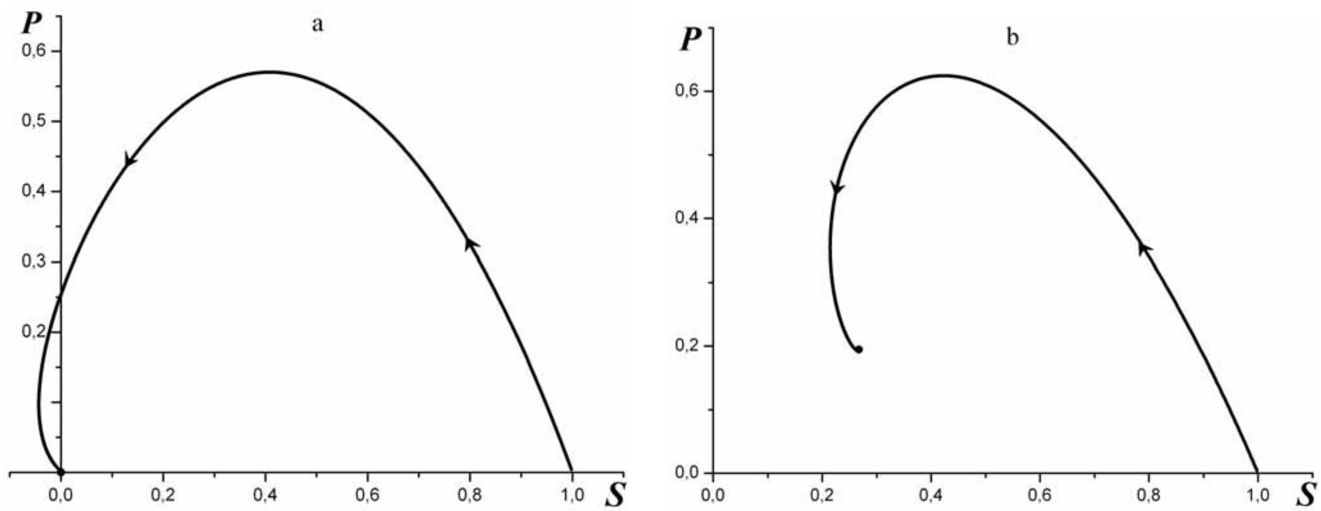

Fig. 1. Phase portraits of the quantum-dot superradiance at $\gamma_P = 10^{-1}$, $\gamma = 2\times10^{-1}$, $\omega = 1$, $F = 10^{-2}$, $g = 1$, $S_e = 1$, and $\gamma_S = 10^{-3}$ (a), $\gamma_S = 10^{-1}$ (b)

population difference do not depend on the time. In this case, system (12), (13) results in the equations

$$P = \frac{\gamma_S}{\gamma_P}\left(S_e - S\right)S, \qquad (15)$$

$$\left(1 - S\right)\left(S_e - S\right) = \frac{\Gamma(S)}{\gamma_S}S, \qquad (16)$$

where the effective attenuation decrement $\Gamma = \Gamma(S)$ is given by expression (14). In the general case, these equations cannot be solved analytically due to the complicated form of the dependence $\Gamma(S)$. The boundary conditions of the weak and strong detunings of the resonator

$$\Gamma \approx \gamma + \begin{cases} \frac{|F|^2(1-S)}{\gamma_P\left[(1-S)^2+g^2S^2\right]}, & |\omega-1| \ll \gamma_{P,S}S_e, \\ \frac{\gamma_P|F|^2}{(\omega-1)^2}\left(1-S\right), & |\omega-1| \gg \gamma_{P,S}S_e \end{cases} \qquad (17)$$

show that the estimate $\Gamma(S) \approx \gamma$ always can be used due to the condition $|F|^2 \ll \gamma_P$. As a result, the stationary values of the population difference and the polarization are given by the expressions

$$S_0 \approx p_+ - \sqrt{p_+^2 - S_e},$$

$$P_0 \approx \frac{\gamma_S}{\gamma_P}\left(-p_- + \sqrt{p_+^2 - S_e}\right)\left(p_+ - \sqrt{p_+^2 - S_e}\right), \qquad (18)$$

where $p_\pm = \frac{1}{2}\left[\left(1 + \frac{\gamma}{\gamma_S}\right) \pm S_e\right]$. Taking the dependence $\Gamma(S)$ into account demonstrates that, with increase in the detuning $|\omega - 1|$, the quantity S_0 grows weakly. The lower the decrement γ_S, the stronger the change of S_0. Moreover, the less the value of γ_P, the stronger the stationary polarization P_0 falls. Since $\gamma_S \ll \gamma_P \lesssim \gamma$, the stationary values are presented by the estimates

$$S_0 \approx \frac{\gamma_S}{\gamma}S_e, \qquad P_0 \approx \frac{\gamma_S^2}{\gamma\gamma_P}S_e^2, \qquad (19)$$

by demonstrating that, in the stationary state of quantum dots, the population difference is much less than the pumping level, while the polarization appears proportional to its square with a still lower coefficient.

In order to analyze the stability of the stationary state (19), it is convenient to put down Eqs. (12), (13) in the symbolic form $\dot{x}_\alpha = f_\alpha$, $\alpha = 1, 2$, where the point marks the time differentiation, the coordinates x_α correspond to the polarization P and the population difference S, while the forces f_α are given by the right-hand sides of the indicated equations. Then the stability of the stationary state $x_\beta = x_{\beta 0}$ is determined by the Jacobi matrix [15]

$$\Lambda_{\alpha\beta} = \left.\frac{\partial f_\alpha}{\partial x_\beta}\right|_{x_\beta = x_{\beta 0}}, \qquad \alpha, \beta = 1, 2, \qquad (20)$$

taking the following form in the case of Eqs.(12) and (13):

$$\hat{\Lambda} \approx \begin{pmatrix} -2\gamma_P & 4\gamma_S S_e \\ -\gamma_P & -\left(\gamma + \gamma_S\right) \end{pmatrix}. \qquad (21)$$

In a vicinity of the stationary state, the law of motion is expressed by the exponential dependence $x_\alpha - x_{\alpha 0} \propto$

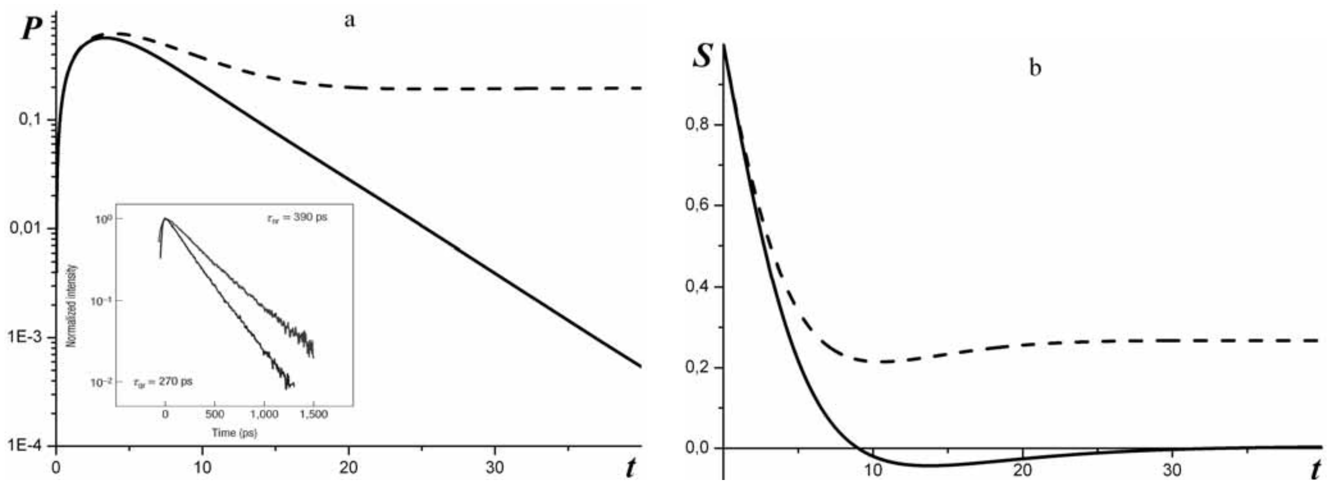

Fig. 2. Time dependences of the polarization (a) and the population difference (b) measured in units of ω_0^{-1} at $\gamma_P = 10^{-1}$, $\gamma = 2\times10^{-1}$, $\omega = 1$, $F = 10^{-2}$, $g = 1$, and $S_e = 1$ (solid lines correspond to $\gamma_S = 10^{-3}$, dashed lines – to $\gamma_S = 10^{-1}$); the inset in Fig. 2,a present the polarization pulses obtained experimentally in [8] (the lower peak corresponds to the resonance conditions, the upper one -– to the absence of a resonance)

$e^{\lambda t}$, whose substitution into the equations of motion (12) and (13) yields the Lyapunov factors $\lambda_P \approx -2\gamma_P$, $\lambda_S \approx -\gamma$. Their negative values testify to the fact that the stationary state (19) corresponds to the attractive node of the phase plane, which results in the stability of the quantum-dot superradiance relative to fluctuations of the polarization and the population difference.

The evolution of the system is described by the phase portraits given in Fig. 1. They demonstrate that, at the real relation between the parameters $|F|^2 \ll \gamma_P$, $\gamma_S \ll \gamma_P \lesssim \gamma$ and in the presence of the pumping S_e, the quantum-dot superradiance is realized in the giant-pulse mode [16] independent on the detuning of the resonator $\omega - 1$ and the coupling parameters F and g. It is significant that an increase of the attenuation parameter of the population difference up to anomalously high values $\gamma_S \sim \gamma_P$ results only in a slight nonmonotonicity of the variation of P and S and a rise of their stationary values (19). But the attenuation oscillations discovered in [13] do not manifest themselves in this case.

Figure 2 presents the time dependences of the polarization $P(t)$ and the population difference $S(t)$ that correspond to the phase portraits in Fig. 1. These dependences confirm the conclusion that an increase of the attenuation parameter of the level populations to anomalously high values $\gamma_S = \gamma_P = 10^{-1}$ results in a slight nonmonotonicity in the variation of the polarization and the population difference and an increase of their stationary values. As concerns the vibrational dependence given in Fig. 3 of work [13], we did not manage to reproduce it, by choosing various values of γ_S, γ_P, γ, ω, F, g, and S_e (except for anomalously large values of the attenuation coefficient γ_S, which can be due to the choice of the coupling parameter F designated in [13] as ν_1 and the frequency ω, whose values are not specified in [13]).

4. Conclusions

The performed phase analysis demonstrates that, in the presence of the pumping $S_e \neq 0$ and the relations between the parameters $|F|^2 \ll \gamma_P$, $\gamma_S \ll \gamma_P \lesssim \gamma$ realized in a quantum-dot ensemble, the superradiance is realized in the form of a giant pulse regardless of the resonator detuning $\omega - 1$ and the coupling parameters F and g. This conclusion is confirmed by the experimental data of work [8], where the authors registered only single pulses, whose form coincided with that obtained in our study (Fig. 2,a).

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ФАЗОВИЙ АНАЛІЗ КОГЕРЕНТНОГО ВИПРОМІНЮВАННЯ АНСАМБЛЮ КВАНТОВИХ ТОЧОК

І.О. Шуда

Р е з ю м е

Проведено фазовий аналіз динамічних рівнянь, отриманих у роботі [13] на основі мікроскопічного подання поляризації ансамблю квантових точок і різниці заселеностей електронних рівнів. Показано, що за наявності накачки та співвідношення параметрів, що реалізуються в ансамблі квантових точок, когерентне випромінювання протікає у режимі гігантського імпульсу незалежно від розладу резонатора і параметрів зв'язку. Проведено порівняння з експериментальними даними.